Mathematical Modelling, Vol. 4, pp. 567–587, 1983
0270-0255/83 $3.00 + .00

THEORIES, MODELS, AND HUMAN-MACHINE SYSTEMS

KENNETH H. FUNK

Department of Industrial and General Engineering
Oregon State University
Corvallis, OR 97331

Abstract—Theories and models play important roles in the development and refinement of knowledge. This paper distinguishes between the two terms and illustrates them. A theory of the operator in human-machine systems, posed in the predicate calculus, is developed. Six axioms address multitask behavior and the response to events. A model of the theory is defined set-theoretically. Issues and implications are briefly explored.

INTRODUCTION

Scientists and engineers seek to acquire and refine knowledge for the purposes of understanding, prediction, and control. They use theories and models as important tools in this process.

The purpose of this paper is twofold. First, it draws a distinction between theories and models and briefly describes the role they take in knowledge refinement. The major portion, however, is concerned with the mechanics of that process. In it is developed a theory of the operator in human-machine systems. The theory is stated in the first-order predicate calculus to provide a wide scope yet to allow precise, unambiguous statements to be made. Next, a model of the theory, a bicycle rider, is defined. Finally, issues and implications raised by the theory and its model are briefly discussed.

THEORIES AND MODELS

While the engineer is usually somewhat more pragmatic than the scientist, they are both vitally interested in the development and refinement of knowledge. Theories and models play an important role in these processes.

A theory is a collection of statements about a subject domain. This definition is consistent with those of Achinstein [1], Chang and Keisler [2], and Suppes [3]. The statements of a theory may be posed in a natural language such as Chinese or French, a purely formal language such as FORTRAN or the first order predicate calculus, a mathematical language such as those of differential equations or probability theory, or some combination of these. The empirical theories of most scientists and engineers are usually formulated in a combination of a natural language and the language of an appropriate form of mathematics. Of course, the language chosen in which to make the statements of the theory will have an important effect on the utility of the theory as well as its acceptance by the scientific and engineering communities.

Logicians, such as Tarski [4], identify important theory components as primitive or undefined terms, defined terms, axioms (the basic assumptions of a theory), and theorems. In deductive theories, theorems result from the application of a logical calculus (rules of logic) to the axioms. A deductive theory is closed under its logical calculus. Empirical theories, while not independent on deductive logic, rely heavily on observation and inductive logic for their theorems.

The term "model" is very important to engineers and scientists and it appears in many seemingly contradictory contexts. Kaplan [5] and Suppes [6] both present surveys of the meanings and uses of the term in the scientific and mathematical literature.

For example, a physical analog is often called a model. A model airplane or the physical analog of a molecule both preserve known and presumed properties of the systems they represent. Similarly, conceptual models, like the physicist's imaginary mass and spring, preserve important properties of systems of interest. Most problematic, though, is the use of the term model to refer to any precisely stated (e.g. mathematical) description of a subject domain. For example, a set of differential equations used to describe the number of inhabitants in an urban area is often referred to as a population model.

Suppes [6] raises the natural question concerning whether or not these widely varying senses of the term model can be reconciled. He concludes that, by taking the perspectives of the logicians (Tarski [4]) and the model theorists (Chang and Keisler [2]) there is a unifying definition.

Quite simply, a model of a theory is a structure in which the statements of the theory are interpreted as true. If a theory describes the physical characteristics of an airplane, an accurate scale model of that airplane will satisfy the theory. A map is a model of a theory of a region of land. A computer simulation can be a model of a theory of population dynamics posed in differential equations. Note that by this definition, if a theory speaks about a system, the system itself is a model of that theory. Przelecki [7] calls this the proper or intended model of the theory.

Theories and models can have an important impact on knowledge development and refinement. The first step in this process of inquiry is to obtain relatively unstructured information about the subject domain. This is then organized into a theory, using a suitable language. Next, models of the theory are studied. These models may include the subject domain itself, they may include other existing structures, or they may be constructed by the researcher, as are simulation models. In any case, these models must satisfy at least portions of the theory.

If, through the study of these models, it is determined that some statements of the theory are violated, one of two conclusions must be drawn. Either the model is wrong (in which case it would not really be a model) or the theory is wrong. Also, by directly comparing other models with the subject domain, similar discrepancies can be observed which must lead to the same alternative conclusions.

By multiple passes through these steps, knowledge is refined. Statements of the theory are dropped and replaced. Occasionally the entire theory must be replaced. One can see evidence of the process in many examples from the history of science.

Since a theory does play such a crucial role in the process of inquiry, the selection of an appropriate language is essential. While most considerations are highly situation-dependent, two general observations can be made. First, the language must be general enough to speak about all relevant aspects of the subject domain. A language chosen solely for its convenience or familiarity may be too constrained to address issues which play an important role in the area of interest.

Equally important is the precision of the language. A theory which is ambiguous can lead to a plethora of models, many of which may not capture the essential features of the subject domain at all. Study of these models in general cannot lead to a better understanding of that subject domain but may in fact misdirect the research and result in erroneous statements.

The remainder of this paper will briefly describe a subject domain, select a language for a theory, pose the foundations for that theory, and present a model of it. The theory will address the operator in human-machine systems.

HUMAN-MACHINE SYSTEMS

A human-machine system (HMS) is a collection of humans and machines which function together to achieve a goal or objective which could not be achieved consistently, efficiently, or safely by any of the components independently. Examples of HMS's include driver-automobile systems, aircrew-aircraft systems, air traffic control systems, nuclear reactor control rooms, command and control systems, and automated office systems.

In any HMS, the human operator plays an important role or he/she would not be there. Although automation has replaced many humans in such systems, there are some functions which machines are simply not yet capable of performing. This is partly due to some of the unique characteristics of human operators.

First, the human operator, unlike most machines, is capable of performing a wide variety of tasks, often in parallel. The human is very good at detecting important events and responding to them in a timely manner. While the human has a limited channel capacity and cannot handle infinitely many tasks simultaneously, he/she is capable of prioritizing tasks and, at any moment, performing those operations that are most crucial to the objectives of the system. Finally, the human is a good decision-maker and planner. Based upon the objectives of the HMS, the human can lay out strategies and identify the sequences of operations that must be performed to achieve the desired outcomes. As long as machines remain unable to do these things efficiently and consistently, there will continue to be human-machine systems.

World War II served as the stimulus for an ever-increasing interest in the human operator in human-machine systems. It was realized then, as it is realized today, that the human is a system component in many very important systems and the performance of the system as a whole depends to a great extent on the performance of the operator. Consequently, quite a bit of effort has gone into developing a better understanding of the human operators in HMS's. A wide variety of theories of the human operators has resulted. Several of the more important ones will be considered now.

One of the earliest systematic approaches to understanding and predicting human performance in HMS's was based on information theory. Researchers, including Deininger and Fitts [8], Fitts and Posner [9], Elkind and Sprague [10], and Senders [11], sought to explain human performance in terms of information transmission. Their efforts led to guidelines for instrument panel layouts and other design issues.

Since many of the systems that humans control are linear in much of their behavior, control engineering theories were developed which described the operator as a servo-mechanism. The McRuer and Jex crossover model [12] described the human operator as a good servo, one whose gain is greater than unity for input frequencies less than the crossover frequency and less than unity for higher frequencies. This and similar linear and quasilinear theories were based primarily on classical control theory. One of the major manual control results to come out of modern control theory was the description of the operators in HMS's as an optimal controller. Developed by Baron and Kleinman [13], the optimal control model assumes that the human operator is well-trained and therefore performs as an optimal servomechanism.

Decision analysis methods were used by researchers, including Cohen and Ferrell [14], to describe the oeprator's decision-making behavior. Typically, decision analysis theories consider the control alternatives the operator has to choose from and how those alternatives are selected.

The methods of queueing theory have found wide application in describing the human operators. Earlier research concentrated on describing visual sampling of instruments as a process in which instruments queue up for visual service. Carbonell [15] and Carbonell *et al.* [16] present the results of such research. The queueing idea has also been applied to multitask situations by Walden and Rouse [17]. Their theory states that tasks queue up for sevice by the operator.

Discrete event theories of the human operators have served as the basis for Monte Carlo simulation models of human-machine systems. In such simulations, such as those described by Siegel and Wolf [18], events occur at random intervals and are processed according to service time distributions. Another application of the discrete event idea is through the use of Petri nets. Petri nets are directed bipartite graphs that have been widely used to describe systems performing sequential and concurrent activities. Peterson [19] provides an excellent survey of Petri nets. Schumacher and Geiser [20] use Petri nets to describe the human operator in multitask situations.

A number of efforts have been made to integrate two or more of the above theoretical perspectives to more adequately address the wide scope of human operator capabilities. Muralidharam and Baron [21] combined decision analysis and optimal control to describe the human operator in a remotely piloted vehicle system. A somewhat more abstract perspective was developed by Johannsen and Rouse [22]. In their theory, the human operator is likened to a time-sharing computer system in which programs compete for computer system resources. The authors discuss how such a structure could be used to integrate a number of more specific methodologies.

Most of the above theories of the human operator address very specific issues in very specific HMS's. In other words, each theory is interpreted in only a very small class of models. Information-theoretic descriptions of the human operator address only information content and not knowledge organization or attention allocation. Optimal control theories cannot explain decision-making behavior. Discrete event theories make statements only about when events occur and how long it takes to process them, not about how they are processed. Consequently, while such theories have been quite successful in addressing the specific issues they were developed to address, they are generally not applicable to the diversity of behavior the human operator exhibits in most operational situations.

Johannsen and Rouse [22] recognized this. They state that "... the success of models in limited domains has not had substantial impact in complex domains ... designers have been known to claim that mathematical models of human behavior are not particularly useful ..." (Note that Johannsen and Rouse use the term model in the sense that the term theory is used in this paper.) Their research is an important step in the effort to remedy the situation. Unfortunately, while their time-sharing computer theory is very wide in scope, it lacks the precision necessary to raise and address specific issues. In other words, the language in which the theory is constructed is too ambiguous to determine what structures do in fact qualify as models. The theory may apply to every HMS or it may apply to none.

There is a need for a theory of the human operator rich enough to consider the human's complex behavior in diverse situations, yet precise enough in language to be able to address specific theoretical and practical issues. The foundations for a candidate theory are presented in the next section.

A THEORY OF THE HUMAN OPERATOR

For a theory to be useful in the process of scientific inquiry, it must be stated in a language general enough to admit its interpretation by a large class of structures in the subject domain, yet it must be precise enough in speaking about that subject domain so that meaningful issues can be posed and addressed. To capture both generality and precision, the predicate calculus with set notation and some supplemental notation will be used. A brief review of set theory and the predicate calculus follows. For additional background, see Stoll [23].

A set is a collection of entities, each entity being called a member or an element of the set. If A is a set containing the elements a, b, and c, the following notation is used:

$$A = \{a, b, c\}$$

If x is an element in A, then $x \in A$ may be written. The set containing no members is called the null or empty set and is denoted $\emptyset$.

From the propositional and predicate calculi, the following notation will be used:

$(\forall x)$	For all x
$(\exists x)$	There exists an x or for some x
$(\exists !)$	There exists exactly one x
$A \Rightarrow B$	If A then B or A implies B
$A \Leftrightarrow B$	A if and only if B
$A \wedge B$	A and B
$A \vee B$	A or B
$\neg A$	Not A
$\ni$	Such that

Occasionally, additional notation will be used with the quantifiers. For example:

$$(\forall x \in A \ni x \in B)$$

could be read "For all x (where x is an element of A) such that x is also an element of B...".

The concepts of the subset relation, set union, set intersection, and relative complement come from the set calculus and may be formally defined.

$$A \subseteq B \Leftrightarrow (\forall x)(x \in A) \Rightarrow (x \in B)$$

$$A \cup B = \{x | (x \in A) \vee (x \in B)\}$$

$$A \cap B = \{x | (x \in A) \wedge (x \in B)\}$$

$$A - B = \{x | (x \in A) \wedge \neg (x \in B)\}$$

In English, A is a subset of B if and only if for all x, if x is an element of A then x is an element of B. The union of the sets A and B is the set consisting of all elements x such that x is an element of A or x is an element of B. The intersection of sets A and B is the set consisting of all elements x such that x is an element of A and x is an element of B. The relative complement of A less B is the set consisting of all elements x such that x is an element of A and x is not an element of B.

The power set of a set, A, denoted

$$\mathscr{P}(A)$$

is the set of all subsets of that set.

A Cartesian product of two sets is the set of all ordered pairs in which the first element in the pair is from the first set and the second element is from the second set:

$$A \times B = \{(a, b) | (a \in A) \wedge (b \in B)\}.$$

A relation is a subset of a Cartesian product:

$$R \subseteq A \times B$$

$$A = \{1, 2, 3\}$$

and

$$\leq \subseteq \mathrm{A} \times \mathrm{A}$$
$$\leq = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}.$$

If R is a relation and $(a, b) \in R$ the notation aRb is often used. Therefore, since $(1, 3) \in \leq$, $1 \leq 3$.

A function is a relation in which each element in the first set is related to only one element in the second set. That is if

$$F \subseteq A \times B,$$

F is a function if and only if

$$(\forall a)(\forall b)(\forall b')((a, b) \in F) \wedge ((a, b') \in F) \Rightarrow (b = b').$$

If this is true, the notation

$$F\colon A \to B$$

is used and read "F maps A to B". If $(a, b) \in F$ then the notation

$$F(a) = b$$

is quite common.

These definitions generalize readily. For example a relation R or a function F could be defined such that

$$R \subseteq A_1 \times A_2 \times \cdots \times A_m$$

or

$$F\colon A_1 \times A_2 \times \cdots \times A_m \to B_1 \times B_2 \times \cdots \times B_n.$$

When a Cartesian product is defined on multiple indexed sets, as for R and F immediately above, a special form of notation may be used:

$$A = A_1 \times A_2 \times \cdots \times A_m = \times \{A_i | i \leq m\}.$$

The theory of the human operator to be developed below is based on the assumption that a human operator in a human-machine system may also be described as a system. The following system definitions are based on those of Mesarovic and Takahara [24], Wymore [25], and Windeknecht [26].

A system S is formally defined as the Cartesian product

$$S \subseteq X \times Y,$$

where X is an input set and Y is an output set. Note that in this most general definition S is a relation and not a function. Theefore, without further specifications, S is nondeterministic and totally independent of any concept of time.

A dynamic system is one whose input and output objects (X and Y) are defined on a time set having special properties.

Let T be a time set, linearly ordered by the relation $\leq$. Let $+$ be a binary associative operation defined on T and let 0 be an identity element of T for $+$. The relation $\leq$, the operation $+$, and the element 0 are formally related in the following manners

$$(\forall t', t'' \in T)(t' \leq t'') \Leftrightarrow (\forall t \in T)(t' + t \leq t'' + t)$$
$$(\forall t \in T)(0 \leq t)$$

That is, $+$ preserves the $\leq$ relation and 0 is a least element of T. To complete the discussion of T itself another relation $<$ may be defined on T such that

$$(t < t') <=> (t \leq t') \wedge (t \neq t')$$

Let A be an input alphabet and let B be an output alphabet. X and Y may then be defined:

$$X \subseteq \{x | x\colon T \to A\} \quad \text{and} \quad Y \subseteq \{y | y\colon T \to B\}.$$

X and Y are sets of input and output segments (or sequences) respectively. Then

$$S \subseteq X \times Y$$

is a dynamic system since its input and output objects are defined on a time set. Elements of S are called behaviors. If $s = (x, y) \in S$ then $x(t)$ and $y(t)$ are the input and output at time t in behavior s of S.

It is occasionally necessary to restrict input and output objects, and even systems, to time segments. The following notational conventions are used:

$$T_{t'} = \{t | t' \leq t\}, \quad T^{t'} = \{t | t < t'\}, \quad T_{t'}^{t''} = \{t | t' \leq t < t''\}.$$

For all $x \in X$

$$x_{t'} = x | T_{t'}, \quad x^{t'} = x | T^{t'}, \quad x_{t'}^{t''} = x | T_{t'}^{t''}.$$

Corresponding notation may be used for Y, the output object and S, the system itself.

Suppose that A is a set of input symbols that a human operator can receive and sense and that B is a set of output symbols that the operator can produce. Let X and Y be defined on a suitable time set as before. Then the operator in a human-machine system may be defined

$$H \subseteq X \times Y.$$

But, again, H is not necessarily a function. In other words, without further specifications, one cannot in general predict the output produced by a human operator defined this way given a particular input. Formally, all that is required to correct this situation is an initial state object. Let C be a set of states. Then H may be defined as a function

$$H\colon C \times X \to Y$$

and knowing initial state and input, output may be identified.

It is more meaningful to carry the notion of operator state beyond just the initial condition. Let Z be defined

$$Z \subseteq \{z | z\colon T \to C\}.$$

Then H may be redefined

$$H: Z \times X \rightarrow Y,$$

and if $h = (z, x, y) \in H$ then $z(t)$ is the state of H at time t in behavior h. z is called a state trajectory.

It is important to point out that while only single-dimensional input, output, and state objects have been discussed, the definitions gereralize to the multidimensional case. That is, if

$$A = A_1 \times A_2 \times \cdots \times A_m, \quad B = B_1 \times B_2 \times \cdots \times B_n, \quad C = C_1 \times C_2 \times \cdots \times C_k,$$

then $x_2(t)$, $y_{13}(t)$, and $z_{27}(t)$ are input component 2, output component 13 and state component 27 at time t.

Practically speaking, the function H may never be completely known. Instead, it may be necessary to generate elements of H, given initial states and segments or sequences of inputs. Ideally, the formal structures for doing this would correspond in a relatively straightforward manner with the way in which human operators actually organize their activities. For that reason, and since human operators frequently organize their activities through the use of procedures, a formal definition for a procedure will be given.

Let

$$H: Z \times X \rightarrow Y$$

be a human operator where

$$Z \subseteq \{z \mid z: T \rightarrow C\}$$

and

$$C = C_1 \times C_2 \times \cdots \times C_k.$$

Let $J \subseteq \{1, 2, 3, \ldots, k\}$ and let $C' = \times \{C_j \mid j \in J\}$. Then a procedure P may be defined

$$P = \{P_{t'}^{t''} \mid (t', t'' \in T) \wedge (t' < t'') \wedge (p_{t'}^{t''}: C \times X_{t'}^{t''} \rightarrow C'\}.$$

That is, P is a family of state transition functions which, given the state of H at the beginning of a time interval and the input over that interval, tell the values of certain, selected state components after that time interval. For all $j \in J$, procedure P controls state component j. This will be denoted $P\langle j \rangle$ (P controls j).

To allow certain further specifications to be made, a consistency condition must be imposed on members of the family of function, P.

$$(\forall t, t_1, t_2, t_3, t_4 \in T)(t_1 + t = t_2) \wedge (t_3 + t = t_4) \Rightarrow (\forall c \in C)(\forall x \in X)(\forall x' \in X)(\forall t' < t)$$
$$((x(t_1 + t') = x(t_3 + t')) \Rightarrow (p_{t_1}^{t_2}(c, x_{t_1}^{t_2}) = p_{t_3}^{t_4}(c, x'^{t_4}_{t_3}))).$$

In other words, P is time invariant. An initial state and input segment will always result in the same final state regardless of when the actions occur.

One additional bit of notation must be introduced here. It will occasionally be necessary to speak of particular state components which P controls. Let $C' = x(C_j \mid j \in J)$ as before and let $c' \in C'$. Then $(q \dot{\in} c')$ is true if and only if q is the member of c' corresponding in place to that of C_i in the Cartesian product. For example, let

$$C' = C_1 \times C_{12} \times C_{13} \times C_{15}$$

and let $c' = (i, j, k, l)$. Then $(j \overset{12}{\in} C')$.

Procedures allow the generation of state trajectories, but there must be a mechanism for generating outputs as well. An output function Q for H may be defined

$$Q\colon C \to B$$

At any time the function Q gives the output of H based on its current state.

An event is a very specific change of state. An event E may be defined

$$E \subseteq C \times C$$

Let $h = (z, x, y) \in H$. Event E is said to occur at time t (denoted $E\langle t \rangle$) if and only if

$$(\exists t' \in T)(\forall t'' \in T^{t}_{t'})(z(t') = z(t'')) \wedge (z(t) \neq z(t')) \wedge ((z(t'), z(t)) \in E)$$

That is, if H is in state c for a period of time and then transitions to state c' at time t and $(c, c') \in E$, event E has occurred.

The terms system, procedure, output function, and event have been defined and necessary auxiliary notation has been introduced. It is now possible to state the axioms of the theory of the human operator.

Let $H\colon Z \times X \to Y$ be a human operator as before. X and Y may be single or multi-dimensional. The state object, Z, must contain at least four components. That is

$$C = C_1 \times C_2 \times \cdots \times C_k$$

and $k \geq 4$. Let P^* be a set of procedures defined on C and X, let Q be an output function defined on C and B, and let E^* be a set of events defined on C.

P^N will be a set of procedure names in direct correspondence with members of P^*. Members of P^N and P^* will be referred to with the same symbols. Context will clearly indicate whether a procedure (a family of functions) or its name is being referenced. Similarly, E^N will be a set of event names in direct correspondence with members of E^*. For each axiom an informal statement will be made, the axiom will be stated formally using the predicate calculus, then a brief explanation will be given.

Axiom 1

Each state component of H is controlled by one or more procedures.

$$(\forall i \in \{1, 2, \ldots, k\})(\exists P \in P^*)(P\langle i \rangle).$$

To generate behaviors of H, each state component must be computed. Since state components are controlled, and therefore determined, by procedures, there must be at least one procedure to control each state component.

Axiom 2 and 3 require that

$$C_1 = \mathcal{P}(P^N) \qquad C_2 = \mathcal{P}(P^N)$$

Axiom 2

At any time, the first state component contains the name(s) of one or more procedures which control each state component.

$$(\forall h = (z, x, y) \in H)(\forall t \in T)(\forall_i \in \{1, 2, \ldots, k\})(\exists P \in P^N)(P \in z_1(t)) \wedge (P\langle i \rangle)$$

$z_1(t)$ is the set of active procedures at time t. If $P \in z_1(t)$ the procedure P is said to be active at time t, otherwise, P is inactive. The active procedures are just those procedures which can potentially affect system state.

Axiom 3

At any time, each state component is determined by exactly one procedure which is identified by the previous membership of its name in state component 2.

$$(\forall h = (z, x, y) \in H)(\forall t'' \in T)(\forall i \in \{1, 2, \ldots, k\})(\exists! P \in P^N)(\exists t' \in T \ni t' < t'')$$
$$\wedge (\forall t \in T_{t'}^{t''})(P \in z_2(t)) \wedge (P\langle i \rangle) \wedge (z_i(t'') \overset{i}{\in} P_{t'}^{t''}(z(t'), x_{t'}^{t''}))$$

$z_2(t)$ is the set of executing procedures at time t. If $P \in z_2(t)$ then $P \in z_1(t)$ and P is executing at time t. If $P \in z_1(t)$ but $P \notin z_2(t)$ then P is said to be suspended (active but not executing). At any time the executing procedures completely determine the state of H at the next instant of time.

Axioms 4 and 5 depend upon the definitions of state components 3 and 4 as sets of event name/procedure name pairs. That is

$$C_3 \subseteq (E^N \times P^N) \qquad C_4 \subseteq (E^N \times P^N)$$

In other words, for any t, $z_3(t)$ and $z_4(t)$ consist of any number of ordered pairs in which the first element of each pair is an event name and the second element is a procedure name.

Axiom 4

At any time, a procedure is active only if it was initially active or if it was earlier specified in an event/procedure pair and the corresponding event occurred.

$$(\forall h = (z, x, y) \in H)(\forall t \in T)(\forall P \in P^N)(t > 0) \wedge (P \in z_1(t))$$
$$\Rightarrow (P \in z_1(0)) \vee ((\exists t' < t)(\exists E \in E^N)((E, P) \in z_3(t')) \wedge (E\langle t \rangle)$$

Procedures are activated according to a plan contained in the third state component. Elements of the plan specify an event which could occur and a procedure to activate if and when that event does occur. A procedure P_P which controls the third state component ($P_P\langle 3 \rangle$) is called a planning procedure.

Axiom 5

If an event/procedure pair is present in the fourth state component and the event occurs when the procedure is active, the procedure becomes inactive.

$$(\forall h = (z, x, y) \in H)(\forall t' \in T)(\forall E \in E^N)(\forall P \in P^N)((E, P) \in z_4(t') \wedge (E\langle t' \rangle) \wedge (P \in z_1(t'))$$
$$\Rightarrow (\exists t'' \in T \ni t' < t'')(\forall t \in T_{t'}^{t''} - \{t'\})(P \notin z_1(t))$$

State component 4 may be called a deactivation plan. Elements of the fourth state component are, again, event/procedure pairs. If a particular pair is present in the deactivation plan while the procedure is active and if the event occurs, then the procedure is deactivated.

Axiom 6

The executive procedure is a procedure which controls state components 1 and 2. It is always both active and executing at time 0.

$$(\exists P_E \in P^N)((P_E\langle 1\rangle) \wedge (P_E\langle 2\rangle)) \wedge (\forall h = (z, x, y) \in H)((P_E \in z_1(0)) \wedge (P_E \in z_2(0)))$$

Active procedures are those procedures which can potentially affect state. Executing procedures are those procedures which actually determine state. The executive procedure determines which procedures are active and which are executing. Consequently, to generate any behavior, the executive procedure itself must be both active and executing at the beginning of the behavior.

Informally speaking, the executive procedure provides primary control for H. Through the executive actions, the behaviors are determined. These actions involve waiting for events. When an event is detected state component 3 (the plan) is searched. If an event/procedure pair is found in the plan in which the event in the pair is the event that just occurred, the procedure is activated. Similarly, active procedures may be deactivated according to state component 4. Then procedures are chosen to execute from the collection of active procedures specified in state component 1. In general, this may require arbitration since there may be several active procedures which control a given state component. This arbitration takes place according to a priority scheme unique to the particular model of the theory in question. After the executing procedures have been determined the executive resumes waiting for the next event.

The axioms account for a number of properties which operators in human-machine systems possess. First, humans often use written or memorized procedures to direct their activities. As events occur and are detected, different procedures may be executed. When events occur that call for quick attention, lower-priority procedures may be interrupted by higher-priority ones (as in dealing with emergencies). Often, when the higher-priority procedures have been completed, the lower-priority ones may resume. All of this seems to be controlled by a higher-level mental mechanism which corresponds to the executive procedure. This mechanism assesses current conditions and allocates mental resources according to relative importance. In addition, the human operator plans ahead. Events which could occur are considered and actions which must be performed if and when the events occur are noted.

A MODEL OF THE THEORY

The theory of the human operator consists of merely six axioms. It does not address specific operations in specific human-machine systems, but instead consists of a few statements which any model of the theory must satisfy. In keeping with the intentions of the research as well as to illustrate the axioms, a model of the theory will now be developed.

The model chosen is a bicycle rider, the operator of a human-machine system which should be familiar to most readers. The rider of a bicycle typically must engage in several simultaneous activities. He plans ahead, moment-by-moment considers the relative importance of pending actions, and performs these actions accordingly. For the sake of clarity and conciseness, in the model only a limited class of situations will be considered and a number of simplifying assumptions will be made. The goal is to illustrate and to raise issues rather than to produce a mathematical structure of high resolution.

The bicycle rider sees the condition of the road ahead of the bicycle. The road may be clear, there might be some distant obstacle (such as a car blocking the road) or there might be a similar obstacle much closer. He also senses velocity. The bicycle might be stopped, moving slowly, moving at a comfortable cruising speed, or moving fast.

The rider operates hand and foot controls. He may either grasp the handlebars or apply the brakes. He may pedal, pedal hard (to accelerate) or rest his feet on the pedals.

In addition to state components necessary for applying the theory to this situation, another state component, that of mental alertness, will be included. The rider could be in a normal state, a danger state in which a distant object has been sensed and plans must be made to deal with it, a panic state in which the plan must be carried out, a safe state in which the plan was successful, and a recovery state in which actions are taken to resume normal riding.

Seven procedures are used to organize the rider's activities. The vision procedure (VP) senses road conditions and velocity. The planning procedure (PP) determines what to do if some sort of obstacle is detected. The rider uses the acceleration procedure (AP) to increase bicycle velocity and the deceleration procedure (DP) to decrease it. The cruise procedure (CP) maintains a comfortable cruising velocity and the stop procedure (SP) brings the bicycle to a stop. The executive procedure (EP) controls and coordinates the other procedures. Consequently, P^N, the set of procedure names, can be defined

$$P^N = \{\text{VP, PP, AP, DP, CP, SP, EP}\}$$

Several events can occur in the rider's restricted world. A danger event (DE) occurs when a distant obstacle in the road is detected. A panic event (PE) occurs when the bicycle is close enough to an obstacle that some evasive action must be performed. A safe event (SE) occurs when the bicycle has stopped safely. A recovery event (RE) occurs when the object is no longer obstructing the path. A normal event (NE) occurs when conditions (velocity and roadway ahead) return to normal. Therefore E^N, the set of event names, is

$$E^N = \{\text{DE, PE, SE, RE, NE}\}$$

H, the bicycle rider, may now be formally defined. A, the input alphabet, is defined as

$$A = A_1 \times A_2$$

where

$$A_1 = \{\text{clear, obstacle-far, obstacle-near}\},$$
$$A_2 = \{\text{stop, slow, cruise, fast}\}.$$

A is the set of inputs H can receive. A_1 is the set of road conditions. A_2 is the set of velocities that can be sensed. B, the output alphabet, may be defined as

$$B = B_1 \times B_2$$

where

$$B_1 = \{\text{grasp, brake}\},$$
$$B_2 = \{\text{rest, pedal, pedal-hard}\}.$$

B is the set of control actions the rider can produce. B_1 is the set of hand control actions. B_2 is the set of foot control actions.

State is defined as

$$C = C_1 \times C_2 \times \cdots \times C_7.$$

C_1 represents active procedures.

$$C_1 = \mathscr{P}(P^N)$$

C_2 represents executing procedures.

$$C_2 = \mathscr{P}(P^N)$$

Actually, C_2 could be more restricted since subsets of P^N in which conflicting procedures appear cannot be the second state component. C_3 is for activation plans.

$$C_3 = (E^N \times P^N)$$

C_4 is for deactivation plans.

$$C_4 = (E^N \times P^N)$$

C_5 is state of mental alertness as described previously.

$$C_5 = \{\text{normal, danger, panic, safe, recover}\}$$

C_6 and C_7 directly influence hand and foot output, respectively.

$$C_5 = \{\text{grasp, brake}\}$$
$$C_6 = \{\text{rest, pedal, pedal hard}\}$$

The time set

$$T = \{0, 1, 2, \ldots, 299, 300\}$$

representing time in units of 0.1 second will be used. Only behaviors covering 30-second intervals will be considered. Standard integer addition, $+$, and the familiar numerical relations $\leq$ and $<$ will be implicit in the discussion

Input, output, and state objects are defined as

$$X = \{x | x\colon T \to A\} \qquad Y = \{y | y\colon T \to B\} \qquad Z = \{z | z\colon T \to C\},$$

and the bicycle rider is then defined as

$$H\colon Z \times X \to Y.$$

The events may be defined more precisely now. As before, the same symbols will be used to refer to both events and to event names. Context will clearly determine meaning.

A danger event (DE) occurs when the rider's mental alertness goes to danger.

$$\text{DE} = \{(c, c') | (c_5 \neq \text{danger}) \wedge (c'_5 = \text{danger})\}$$

The panic event (PE), safe event (SE), recover event (RE), and the normal event (NE) are defined analogously as

$$\mathrm{PE} = \{(c, c') | (c_5 \neq \text{panic}) \wedge (c'_5 = \text{panic})\}$$

$$\mathrm{SE} = \{(c, c') | (c_5 \neq \text{safe}) \wedge (c'_5 = \text{safe})\}$$

$$\mathrm{RE} = \{(c, c') | (c_5 \neq \text{recover}) \wedge (c'_5 = \text{recover})\}$$

$$\mathrm{NE} = \{(c, c') | (c_5 \neq \text{normal}) \wedge (c'_5 = \text{normal})\}.$$

Recall that procedures are families of state transition functions that affect specific state components. For each of the bicycle rider model's procedures, the purpose of the procedure will first be given. Next, a formal description of the family of functions will be presented. Since T is a discrete time set, it is possible to define each of the functions as one-step state transition functions. Then a formal predicate calculus description of the procedure will be given. Finally, the procedure will be described informally.

The vision procedure, VP, senses the condition of the road ahead and the velocity of the bicycle. VP controls state component 5, the bicycle rider's mental alertness.

$$\mathrm{VP} = \{vp_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (vp_t^{t+1}: C \times X_t^{t+1} \rightarrow C_5)\}$$

$(\forall h = (z, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{VP} \in z_2(t)) \Rightarrow$

$$(z_5(t) = \text{normal}) \wedge (x_1(t) \neq \text{clear}) \Rightarrow (z_5(t+1) = \text{danger})$$

$$(z_5(t) = \text{danger}) \wedge (x_1(t) = \text{obstacle-near}) \Rightarrow (z_5(t+1) = \text{panic})$$

$$(z_5(t) = \text{panic}) \wedge (x_2(t) = \text{stop}) \Rightarrow (z_5(t+1) = \text{safe})$$

$$(z_5(t) = \text{safe}) \wedge (x_1(t) = \text{clear}) \Rightarrow (z_5(t+1) = \text{recover})$$

$$(z_5(t) = \text{recover}) \wedge (x_2(t) = \text{cruise}) \Rightarrow (z_5(t+1) = \text{normal})$$

Otherwise, $z_5(t+1) = z_5(t)$.

Informally, for any behavior of the bicycle rider, at any time (except the final time value, since the next time value would then be undefined), if the vision procedure is executing, the following is true. If the rider is in the normal state of mental alertness and an obstacle appears in the road ahead, at the next time instant the rider will be in the danger state of mental alertness. If, while in the danger state, the bicycle closely approaches such an obstacle, the rider goes to the panic state. If the bicycle comes to a stop while the rider is in the panic state, he goes to the safe state. Then when the road becomes clear again he goes to the recover state. Finally, when a comfortable cruise velocity is again reached, the rider returns to the normal state. If none of the above conditions apply, his state remains unchanged.

The planning procedure, PP, stores a plan to stop the bicycle in state component 3 and a plan to deactivate the required procedures in state component 4. In addition, it removes entries from these state components as events occur so that procedures will not be prematurely activated or deactivated later.

$$\mathrm{PP} = \{pp_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (pp_t^{t+1}: C \times X_t^{t+1} \rightarrow C_3 \times C_4)\}$$

$(\forall h = (z, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{PP} \in z_2(t) \Rightarrow$

$$z_3(t) = \{(\mathrm{DE}, \mathrm{PP})\} \Rightarrow$$
$$z_3(t+1) = \{(\mathrm{PE}, \mathrm{DP}), (\mathrm{PE}, \mathrm{SP}), (\mathrm{RE}, \mathrm{AP})\}$$
$$z_4(t+1) = \{(\mathrm{SE}, \mathrm{DP}), (\mathrm{RE}, \mathrm{SP}), (\mathrm{NE}, \mathrm{AP}), (\mathrm{NE}, \mathrm{PP})\}$$
$$z_3(t) \neq \{(\mathrm{DE}, \mathrm{PP})\} \Rightarrow$$
$$z_3(t+1) = z_3(t) - \{(E, P) | (E\langle t \rangle) \wedge (P \in z_1(t))\}$$
$$z_4(t+1) = z_4(t) - \{(E, P) | (E\langle t \rangle) \wedge (P \in z_1(t))\}$$

When the planning procedure first begins executing, the entry (DE, PP), which brought about its activation, will be in state component 3. Then PP will store entries in state components 3 and 4 to bring the bicycle to a stop if conditions deteriorate. A panic event (PE) will cause the activation of both the deceleration and the stop procedures (DP and SP). A recovery event (RE) will cause the activation of the acceleration procedure (AP). In addition, a safe event (SE) will cause the deactivation of the deceleration procedure. A recovery event will cause the deactivation of the stop procedure (SP). A normal event (NE) will cause the deactivation of both the acceleration procedure (AP) and the planning procedure (PP) itself. In addition, PP removes from state components 3 and 4 entries specifying events that have just occurred.

The acceleration procedure, AP, is used to increase the velocity of the bicycle. It controls state components 6 and 7.

$$\mathrm{AP} = \{ap_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (ap_t^{t+1}: C \times X_t^{t+1} \rightarrow C_6 \times C_7)\}$$

$$(\forall h = (z, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{AP} \in z_2(t)) \Rightarrow$$

$$z_6(t+1) = \text{grasp}$$
$$(x_2(t) = \text{stop}) \Rightarrow (z_7(t+1) = \text{pedal-hard})$$
$$(x_2(t) = \text{slow}) \Rightarrow (z_7(t+1) = \text{pedal-hard})$$
$$(x_2(t) = \text{cruise}) \Rightarrow (z_7(t+1) = \text{pedal})$$
$$(x_2(t) = \text{fast}) \Rightarrow (z_7(t+1) = \text{pedal}).$$

While accelerating, the rider grasps the handle bars. If the bicycle is stopped or moving slowly, the rider must pedal hard to bring it up to speed. If the bicycle is moving faster, it is only necessary for the rider to pedal with a comfortable degree of effort.

The cruise procedure, CP, maintains the velocity of the bicycle at a comfortable level. It controls state components 6 and 7.

$$\mathrm{CP} = \{cp_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (cp_t^{t+1}: C \times X_t^{t+1} \rightarrow C_6 \times C_7)\}$$

$$(\forall h = (x, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{CP} \in z_2(t)) \Rightarrow$$

$$z_6(t+1) = \text{grasp}$$
$$(x_2(t) = \text{stop}) \Rightarrow (z_7(t+1) = \text{pedal-hard})$$
$$(x_2(t) = \text{slow}) \Rightarrow (z_7(t+1) = \text{pedal-hard})$$
$$(x_2(t) = \text{cruise}) \Rightarrow (z_7(t+1) = \text{pedal})$$
$$(x_2(t) = \text{fast}) \Rightarrow (z_7(t+1) = \text{rest}).$$

While cruising, the rider grasps the handle bars. If the bicycle stops or is moving slowly, the rider must pedal hard. At cruise velocity, pedalling at a comfortable level of exertion is all that is required. If velocity becomes too great, the rider simply stops pedalling.

The deceleration procedures, DP, is used to slow the bicycle. It controls state components 6 and 7.

$$\mathrm{DP} = \{dp_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (dp_t^{t+1}: C \times X_t^{t+1} \rightarrow C_6 \times C_7)\}$$

$(\forall h = (z, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{DP} \in z_2(t)) \Rightarrow$

$$(x_2(t) = \text{fast}) \vee (x_2(t) = \text{cruise}) \Rightarrow (z_6(t+1) = \text{grasp}),$$

$$(x_2(t) = \text{slow}) \vee (x_2(t) = \text{stop}) \Rightarrow (z_6(t+1) = \text{brake}),$$

$$z_7(t+1) = \text{rest}.$$

While decelerating, the rider grasps the handle bars until the velocity drops below the cruising velocity. Then brakes are applied. No pedalling is done.

The stop procedure, SP, is used to bring the bicycle to a stop. It controls state components 6 and 7.

$$\mathrm{SP} = \{sp_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (sp_t^{t+1}: C \times X_t^{t+1} \rightarrow C_6 \times C_7)\}$$

$(\forall h = (x, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{SP} \in z_2(t)) \Rightarrow$

$$(z_6(t+1) = \text{brake})$$

$$(z_7(t+1) = \text{rest}).$$

While stopping, the rider applies the brakes and does not pedal.

The executive procedure, EP, controls the activation, execution, and deactivation of procedures. It controls state components 1 and 2.

$$\mathrm{EP} = \{ep_t^{t+1} | (t \in T) \wedge (t < 300) \wedge (ep_t^{t+1}: C \times X_t^{t+1} \rightarrow C_1 \times C_2)\}.$$

Let R be the set of procedures which control state components 6 and 7

$$R = \{\mathrm{AP}, \mathrm{CP}, \mathrm{DP}, \mathrm{SP}\}$$

and let $>$ be a priority relation defined on R

$$> \subseteq R \times R$$

such that

$$\mathrm{DP} > \mathrm{SP} > \mathrm{AP} > \mathrm{CP}.$$

That is, the deceleration procedure has the highest priority in the group. Then

$(\forall h = (z, x, y) \in H)(\forall t \in T \ni t < 300)(\mathrm{EP} \in z_2(t)) \Rightarrow$

$$G_t = z_1(t) \cup \{P | (\exists E \in E^N)(E\langle t \rangle) \wedge ((E, P) \in z_3(t))\}$$
$$- \{P | (\exists E \in E^N)(E\langle t \rangle) \wedge ((E, P) \in z_4(t))\}$$
$$W_t = G_t \cap R$$
$$(\exists P' \in P^N)(P' \in W_t) \wedge (\forall P \in W_t)((P' \neq P) \Rightarrow (P' > P))$$
$$z_1(t + 1) = G_t$$
$$z_2(t + 1) = (G_1 - (W_t - \{P'\})).$$

At any time, G_t is the set of active procedures for the next instant of time. It is determined by considering currently active procedures, adding these procedures which are to be activated because of an event which just occurred and removing those procedures to be deactivated because of an event. W_t is the set of those active procedures which control state components 6 and 7. P' is the highest priority procedure in W_t. That is, P' is the highest priority active procedure which controls state components 6 and 7.

The first state component at time $t + 1$ is simply the set G_t. The second state component at time $t + 1$, the executing procedures, consists of the active procedures except that only the highest priority procedure in W_t is allowed to execute. This assures that no more than one procedure ever "tries" to control the rider's hands and feet.

The output function for H is the function Q

$$Q: C \rightarrow B$$

$(\forall h = (z, x, y) \in H)(\forall t \in T)$

$$Q(z(t)) = (z_6(t), z_7(t))$$

That is, the rider's hand output has the same value as that of state component 6. The rider's foot output has the same value as that of state component 7.

To construct a behavior for H, an initial state must be given and a sequence of inputs to H must be provided. The following steps may then be used.

(a) Set $t = 0$.
(b) Determine the state at time $t + 1$ according to the rules given for each procedure.
(c) Determine the output at time t according to the function Q.
(d) Set $t = t + 1$.
(e) If $t \leq 300$, go to step (b).
(f) Otherwise, stop.

An example behavior is given in Tables 1 and 2. Table 1 gives the state trajectory of H in a behavior $h = (z, x, y)$. Table 2 gives input and output for the same behavior.

In this behavior of H at time 0, the executive procedure (EP), the vision procedure (VP) and the cruise procedure (CP) are active and executing. If a danger event (DE) occurs, the planning procedure (PP) will be activated. There is currently no deactivation plan in state component 4. The rider is in the normal state of mental alertness, grasping the handle bars and pedaling at a comfortable rate. The roadway ahead is clear and the bicycle is moving at a comfortable cruise velocity. Output is of course identical to state components 6 and 7.

Conditions remain the same until time 30 (3 seconds) when a distant obstacle (say, a car blocking the road) is sighted ($x_1(30)$). This causes the rider's state of mental alertness to change to danger ($z_5(31)$), which in turn brings about the activation and execution of PP at

Table 1. Bicycle rider state

t	$z_1(t)$	$z_2(t)$	$z_3(t)$	$z_4(t)$	$z_5(t)$	$z_6(t)$	$z_7(t)$
0	{EP, VP, CP}	{EP, VP, CP}	{(DE, PP)}	∅	normal	grasp	pedal
1–29	,,	,,	,,	,,	,,	,,	,,
30	,,	,,	,,	,,	,,	,,	,,
31	,,	,,	,,	,,	danger	,,	,,
32	{EP, VP, CP, PP}	{EP, VP, CP, PP}	,,	,,	,,	,,	,,
33	,,	,,	{(PE, DP) (PE, SP) (RE, AP)}	{(SE, DP) (RE, SP) (NE, AP) (NE, PP)}	,,	,,	,,
34–79	,,	,,	,,	,,	,,	,,	,,
80	,,	,,	,,	,,	,,	,,	,,
81	,,	,,	,,	,,	panic	,,	,,
82	{EP, VP, CP, PP, DP, SP}	{EP, VP, PP, DP}	{(RE, AP)}	,,	,,	,,	,,
83	,,	,,	,,	,,	,,	,,	rest
84–99	,,	,,	,,	,,	,,	,,	,,
100	,,	,,	,,	,,	,,	,,	,,
101	,,	,,	,,	,,	,,	brake	,,
102–119	,,	,,	,,	,,	,,	,,	,,
120	,,	,,	,,	,,	,,	,,	,,
121	,,	,,	,,	,,	safe	,,	,,
122	{EP, VP, CP, PP, SP}	{EP, VP, PP, SP}	,,	{(RE, SP) (NE, AP) (NE, PP)}	,,	,,	,,
123–219	,,	,,	,,	,,	,,	,,	,,
220	,,	,,	,,	,,	,,	,,	,,
221	,,	,,	,,	,,	recovery	,,	,,
222	{EP, VP, CP, PP, AP}	{EP, VP, PP, AP}	∅	{(NE, AP) (NE, PP)}	,,	,,	,,
223	,,	,,	,,	,,	,,	grasp	pedal-hard
224–228	,,	,,	,,	,,	,,	,,	,,
229	,,	,,	,,	,,	,,	,,	,,
230–269	,,	,,	,,	,,	,,	,,	,,
270	,,	,,	,,	,,	,,	,,	,,
271	,,	,,	,,	,,	normal	,,	pedal
272	{EP, VP, CP}	{EP, VP, CP}	,,	∅	,,	,,	,,
273–300	,,	,,	,,	,,	,,	,,	,,

time 32 ($z_1(32), z_2(32)$). PP places entries to activate and deactivate the deceleration, stop, and acceleration procedures (DP, SP, AP) in state components 3 and 4 at time 33.

At time 80, the bicycle has come near the obstacle ($x_1(80)$). This causes the rider to panic ($z_5(81)$) and in turn causes DP and SP to be activated. Note that now, while CP, DP, and SP are all active, DP is the only one of the three allowed to execute since it has the highest priority.

Velocity drops to slow at time 100 ($x_2(100)$) and the deceleration procedure begins braking at time 101 ($z_6(101), y_1(101)$) as a result.

At time 120 the bicycle comes to a halt ($x_1(120)$) bringing the rider to the safe state ($z_5(121)$), deactivating DP, and allowing SP to execute ($z_1(122), z_2(122)$).

When the road becomes clear again at time 220 ($x_1(220)$) the rider starts up using AP and eventually returns to cruising speed ($x_2(270)$), after which PP, having completed the overseeing of the evasive actions finishes cleaning up state components 3 and 4 and becomes inactive.

Figure 1 is an active procedure profile of h. It depicts executing procedures (solid lines) and suspended procedures (dashed lines) as functions of time.

Table 2. Bicycle rider input and output

t	$x_1(t)$	$x_2(t)$	$y_1(t)$	$y_2(t)$
0	clear	cruise	grasp	pedal
1–29	,,	,,	,,	,,
30	obstacle-far	,,	,,	,,
31	,,	,,	,,	,,
32	,,	,,	,,	,,
33	,,	,,	,,	,,
34–79	,,	,,	,,	,,
80	obstacle-near	,,	,,	,,
81	,,	,,	,,	,,
82	,,	,,	,,	,,
83	,,	,,	,,	rest
84–99	,,	,,	,,	,,
100	,,	slow	,,	,,
101	,,	,,	brake	,,
102–119	,,	,,	,,	,,
120	,,	stop	,,	,,
121	,,	,,	,,	,,
122	,,	,,	,,	,,
123–219	,,	,,	,,	,,
220	clear	,,	,,	,,
221	,,	,,	,,	,,
222	,,	,,	,,	,,
223	,,	,,	grasp	pedal-hard
224–228	,,	,,	,,	,,
229	,,	slow	,,	,,
230–269	,,	,,	,,	,,
270	,,	cruise	,,	,,
271	,,	,,	,,	pedal
272	,,	,,	,,	,,
273–300	,,	,,	,,	,,

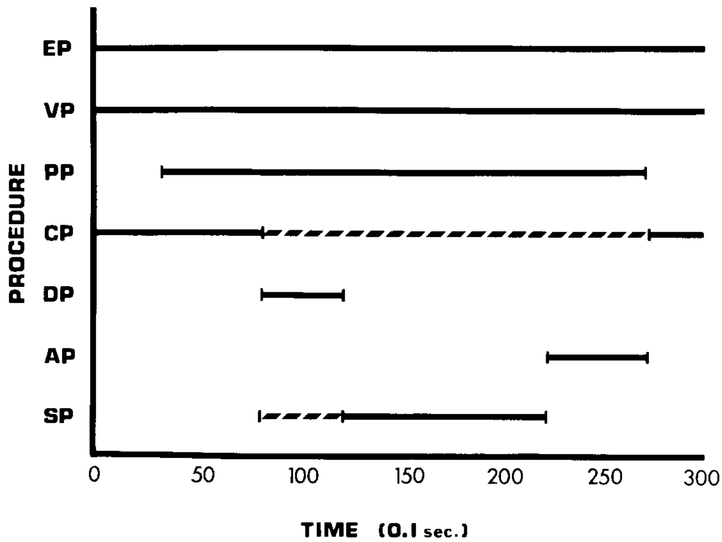

Fig. 1. Bicycle rider active procedure profile.

IMPLICATIONS AND ISSUES

The bicycle-rider model is an admittedly highly simplified one, yet it serves two important purposes. First, it illustrates the general relationship between models and theories. The bicycle rider, as described set-theoretically, satisfies the human operator theory, so it is a model of the theory. Equally important, it illustrates the human operator theory presented in an earlier section.

As a result, several issues raised by the theory become evident. Of particular significance is that the human operator theory provides the potential to integrate several different existing theories of human operators in a single representation. For example, the bicycle-rider model could be extended to include an optimal control procedure for controlling the direction of the bicycle and a decision-making procedure to select from several alternative ways to avoid an obstacle.

The careful reader will note "holes" in the model. Various initial state/input sequence combinations could be developed which would cause the model to exhibit irrational behavior-riding right into an obstacle, for example. But, after all, bicycle riders often do react irrationally, partly because they fail to plan ahead, partly because sometimes things just happen too rapidly for them to cope. The ability of the theory and its model to account for human error is an important advantage.

The theory raises some interesting questions about goals. Although not directly addressed by the axioms, any planning activity must be done with respect to some goal or objective. Future versions of the theory must deal with this concept.

Another matter not directly addressed is that of written procedures. Operators in HMS's often use handbooks to guide them through procedure execution. Although the theory does not include this idea explicitly, it at the same time does not preclude it. Further investigation is required. Similarly, additional research should consider procedure synthesis, in which operators "write" their own procedures to deal with new situations.

In summary, this paper has attempted to distinguish between theories and models and to show that both are important in knowledge development and refinement. A theory and model of the operator of human machine systems were presented. This theory and its models provide a means for understanding and predicting the behavior of the human in this important class of systems.

REFERENCES

1. P. Achinstein, *Concepts of Science*, The Johns Hopkins Press, Baltimore (1968).
2. C. C. Chang and H. J. Keisler, *Model Theory*, Elsevier North Holland Publishing Company, Amsterdam (1973).
3. P. Suppes, A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences, in *The Concept and Role of the Model in Mathematics and Natural and Social Sciences*, D. Reidel Publishing Company (1961).
4. A. Tarski, *Introduction to Logic*, Oxford University Press, New York (1965).
5. A. Kaplan, *The Conduct of Inquiry*, Harper & Row, Publishers, New York (1964).
6. P. Suppes, The Desirability of Formalization in Science, *J. Phil.*, **65**, 651–664 (1968).
7. M. Przelecki, *The Logic of Empirical Theories*, Routledge & Kegan Paul, New York (1969).
8. R. L. Deininger and P. M. Fitts, Stimulus Response Compatibility, Information Theory, and Perceptual Motor Performance, in *Information Theory in Psychology*, H. Quastler, editor, The Free Press, Glencoe, Illinois (1955).
9. P. M. Fitts and M. I. Posner, *Human Performance*, Brooks/Cole Publishing Company, Belmont, California (1969).
10. J. I. Elkind and L. T. Sprague, Transmission of Information in Simple Manual Control Systems, *IRE Trans. Hum. Factors Electron.*, **HFE-2**, No. 1, 58–60 (1961).
11. J. W. Senders, Man's Capacity to Use Information from Complex Displays, in *Information Theory in Psychology*, H. Quastler, editor, The Free Press, Glencoe, Illinois (1955).
12. D. T. McRuer and R. Jex, A Review of Quasi-linear Pilot Models, *IEEE Trans. Hum. Factors Electron.*, **HFE-8** (3), 231–249 (1967).
13. S. Baron and D. L. Kleinman, The Human as an Optimal Controller and Information Processor, *IEEE Trans. Man-Machine Syst.* **MMS-10** (1), 9–17 (1969).

14. H. S. Cohen and W. R. Ferrell, Human Operator Decision Making in Manual Control, *IEEE Trans. Man-Machine Syst.* **MMS-10** (2), 41-47 (1969).
15. J. R. Carbonell, A Queueing Model of Many-Instrument Visual Sampling, *IEEE Trans. Hum. Factors Electron.*, **HFE-7**, (4), 157–164 (1966).
16. J. R. Carbonell, J. L. Ward, and J. W. Senders, A Queueing Model of Visual Sampling: Experimental Validation, *IEEE Trans. Man-Machine Syst.*, **MMS-9** (3), 82–87 (1968).
17. R. S. Walden and W. B. Rouse, A Queueing Model of Pilot Decision Making in a Multitask Flight Management Situation, *IEEE Trans. on Syst. Man. Cybern.*, **SMC-8**, (12), 867–874 (1978).
18. A. I. Siegel and J. J. Wolf, *Man-Machine Simulation Models*, John Wiley and Sons, New York (1969).
19. J. L. Peterson, Petri Nets, *Comput. Surv.*, 223–252 (Sept. 1977).
20. W. Schumacher and G. Geiser, Petri Nets as a Modeling Tool for Discrete Concurrent Tasks, in *Proceedings of the* 14*th Annual Conference on Manual Control*, NASA Ames Research Center, Moffett Field, California (1978).
21. R. Muralidharam and S. Baron, Combined Monitoring, Decision, and Control Model for the Human Operator in a Command and Control Task, in *Proceedings of the* 14*th Annual Conference on Manual Control*, NASA Ames Research Center, Moffett, Field, California (1978).
22. G. Johannsen and W. B. Rouse, Mathematical Concepts for Modeling Human Behavior in Complex Man-Machine Systems, *Human Factors*, **21**, (6), 733–747 (1979).
23. R. R. Stoll, *Set Theory and Logic*, Dover Publications, Inc., New York (1979).
24. M. D. Mesarovic and Y. Takahara, *General Systems Theory*: *Mathematical Foundations*, Academic Press, New York (1975).
25. A. W. Wymore, *Systems Engineering Methodology for Interdisciplinary Teams*, John Wiley & Sons, New York (1976).
26. T. G. Windeknecht, *General Dynamical Processes*, Academic Press, New York (1971).